STATUS REPORT

GRANT NAG8-059

APPLICATION OF SATELLITE DATA IN VARIATIONAL ANALYSIS FOR GLOBAL CYCLONIC SYSTEMS

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1. Introduction

Summaries of work underway during the second year of Grant NAG8-059 are presented in the following sections. Some of the research has progressed sufficiently for the preparation of manuscripts and the status of these is noted. Other ongoing research is not yet ready for publication. A bibliography of all papers and abstracts presented as part of development of the variational objective analysis follows the summaries.

The goal of our research is a variational data assimilation method that incorporates as dynamical constraints, the primitive equations for a moist, convectively unstable atmosphere and the radiative transfer equation. Variables to be adjusted include the three-dimensional vector wind, height, temperature, and moisture from rawinsonde data, and cloud-wind vectors, moisture, and radiance from satellite data. This presents a formidable mathematical problem. In order to facilitate thorough analysis of each of the model components, we defined four variational models that divide the problem naturally according to increasing complexity. The first variational model (MODEL I) contains the two nonlinear horizontal momentum equations, the integrated continuity equation, and the hydrostatic equation.

MODEL II contains MODEL I plus the thermodynamic equation for a dry adiabatic atmosphere. The introduction of this additional constraint violates the requirement that the number of subsidiary conditions (dynamic constraints) must be at least one less than the number of dependent variables (Courant, 1936). Inclusion of the same number of constraints as dependent variables

overdetermines the problem and a solution is not guaranteed. Therefore, we must develop a scheme to circumvent this problem or else the dynamically adjusted meteorological variables will not satisfy the closed set of primitive equations.

MODEL III includes MODEL II plus radiance as a dependent variable and the radiative transfer equation as a constraint. MODEL IV contains MODEL III plus an additional moisture variable, a moisture conservation equation and a parameterization for moist adiabatic processes.

In addition, the variational models will be made more responsive to the original observations. The direct methods for calculating derivatives require that the data be pregridded. As noted by Achtemeier (1975) and Williamson and Daley (1983), pregridding removes the dependence of the final analysis upon the original observations. This dependence can be reestablished by merging the variational models with the successive corrections interpolation method (SCM) through a cyclical procedure. Independently interpolated meteorological variables are merged variationally. The variational fields then serve as first guess fields for the next SCM analysis, and so on.

The research carried out during the second year thus far of the current project has fallen into four areas several of which are ongoing and are described in the first interim report (Achtemeier, 1987). The research areas are 1) sensitivity studies involving MODEL I, 2) evaluation of MODEL II, 3) reformulation of MODEL I for greater compatibility with MODEL II, 4) development of MODEL III (radiative transfer equation), and 5) making the model more responsive to the observations.

Brief summaries of the progress in each research area follow:

1. Sensitivity Studies. After transferral to the ISWS VAX-75 computer, MODEL
I was run with a range of precision modulus weights. The purpose of the study
was to assess the sensitivity of MODEL I to different distributions of weights
through percentage reductions of the initial unadjustment as functions of
cycle through the solution sequence and through pattern recognition of the
resultant fields of adjusted meteorological variables. Preliminary results
show that there exists values for some precision moduli for which MODEL I does
not converge to a solution. Therefore, this variational assimilation places
limits upon observation accuracy. This may impact upon the design of new
instruments and new observation systems.

2. MODEL II Evaluation. After 4 cycles through the sequence of variational
equations, it was found that the RMS differences between the initial
unadjustment and the variational adjustment did not decrease at all levels for
the thermodynamic equation. This problem has been traced to the way the
vertical velocity is calculated in the MODEL I. The development and evaluation
of MODEL II is the subject of the more detailed presentation that follows.

3. Reformulation of MODEL I. It is crucial that there be complete
compatibility between MODEL I and MODEL II. The theoretical conditions that
make the two models compatible are the subjects of the more detailed
presentation that follows. A new version of MODEL I (MODEL 1.2) is under
development.

4. MODEL III (Radiative Transfer Equation). The development of the radiative transfer equation as a variational constraint has proceeded in two ways. First, the radiative transfer equation is posed as a variational constraint and the traditional physical temperature retrieval is treated as an applied variational problem. First guess temperature profiles are required for a solution along with brightness temperatures from any or all of the four microwave channels of the TOVS instrument. Nonlinearities in the radiative transfer equation are retained in the variational formulation through a cyclical adjustment whereby the adjustment equations are solved repeatedly with the nonlinear terms updated with previously adjusted values. Rapid convergence to a solution is obtained in several cycles. Second, the brightness temperatures become four independent variables and the radiative transfer equation becomes a constraint along with the truncated Navier-Stokes equations in a diagnostic variational model for a non-divergent, geostrophic, hydrostatic, and dry atmosphere. This formulation allows the brightness temperatures, normally related to temperature only as a function of height, to be related to temperatures and winds in three dimensions.

5) Coupling Variational Models with Observations. All of the variational assimilation models are derived from linear and nonlinear partial differential equations. It is therefore necessary that the variables to be adjusted be transferred from randomly spaced observing sites to a regular grid in a highly accurate way so that the derivatives of the gridded variable be accurate and free from interpolation error. A detailed theoretical analysis of the Barnes (1964, 1973) objective interpolation method has led to a modification that yields improved higher-order calculations including up to 72 percent reduction of undesirable very short wavelengths in the Laplacian of the height field.

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Project Bibliography

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a) Published Papers

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Achtemeier, G. L., and H. T. Ochs, 1988: A multivariate variational objective analysis - assimilation method. Part 1: Development of the basic model. (Submitted to Tellus)

Achtemeier, G. L., S. Q. Kidder and R. W. Scott, 1988: A variational assimilation method for the diagnosis of cyclone systems. Part II: Case study results with and without satellite data. (Submitted to Tellus)

b) Abstract of new paper

ABSTRACT: The radiative transfer equation is posed as a variational constraint and the traditional physical temperature retrieval is treated as an applied variational problem. Results show that first guess temperature profiles are required for a solution along with brightness temperatures from

any or all of the four microwave channels of the TOVS instrument. Nonlinearities in the radiative transfer equation are retained in the variational formulation through a cyclical adjustment whereby the adjustment equations are solved repeatedly with the nonlinear terms updated with previously adjusted values. Rapid convergence to a solution is obtained in several cycles.

Then the brightness temperatures become four independent variables and the radiative transfer equation becomes a constraint along with the truncated Navier-Stokes equations in a diagnostic variational model for a non-divergent, geostrophic, hydrostatic, and dry atmosphere. This formulation allows the brightness temperatures, normally related to temperature only as a function of height, to be related to temperatures and winds in three dimensions.

MODEL II (Version 1)

1. Introduction

The MODEL II variational data assimilation model is the second of four general assimilation models designed to blend weather data measured from space-based platforms into the meteorological data mainstream in a way that maximizes the information content of the satellite data. Because there are many different observation locations and there are many instruments with different measurement error characteristics, it is also necessary to require that the blending be done to maximize the information content of the data to retain a dynamically consistent and reasonably accurate description of the state of the atmosphere. This is ideally a variational problem for which the data receive relative weights that are inversely proportional to measurement error and are adjusted to satisfy a set of dynamical equations that govern atmospheric processes. Because of the complexity of this type of variational problem, we have divided the problem into four variational models of increasing complexity. The first, MODEL I, includes as dynamical constraints the two horizontal momentum equations, the hydrostatic equation, and an integrated continuity equation. The second, MODEL II includes as dynamical constraints, the equations of MODEL I plus the thermodynamic equation for a dry atmosphere.

MODEL II includes as dynamical constraints the five primitive equations that govern atmospheric flow. The reason for delaying the introduction of the thermodynamic equation until MODEL II is as follows. Courant (1936) showed

that the number of subsidiary conditions (dynamic constraints) must be at least one less than the number of adjustable dependent variables. The five primitive equations form a closed set of equations with five dependent variables. Inclusion of the same number of constraints as dependent variables overdetermines the problem and a solution is not guaranteed. Achtemeier (1975) attempted to circumvent this problem through a parameterization of the tendency terms of the velocity components and the temperature that required the exact solution of the integrated continuity equation. This method, a variational adjustment within a variational adjustment, was considered a failure after an extensive analysis (Achtemeier, 1979) found unrealistically large velocity component tendencies where actual velocity changes over a 12-hr period were small.

The approach taken in the development of MODEL I was to make possible the inclusion of the five primitive equations by increasing the number of dependent variables. We defined two new dependent variables, the developmental components of the horizontal velocity tendencies, which increased the number of dependent variables from five to seven. Though this solves the problem of the number of subsidiary conditions, the extent of internal coupling among the variables and within the equations could not be determined fully until the development and evaluation of MODEL II.

2. MODEL II: Thermodynamic Equation as a Dynamic Constraint

Defluxing and omitting the dissipation term of the thermodynamic equation in Anthes and Warner (1978), the thermodynamic equation as it appears as a dynamical constraint in MODEL II is,

$$\partial T/\partial t + m \left(u \, \partial T/\partial x + v \, \partial T/\partial y\right) + \dot{\sigma} \, \partial T/\partial \sigma - RT\omega/C_p p - Q/C_p = 0 \qquad (1)$$

The omega-term (Term 4) of the thermodynamic equation can be transformed into the nonlinear sigma coordinate system through the definition,

$$\sigma = \beta \left(p-p^*\right)3 + \sigma^* \left(p-p_u\right)/\left(p^*-P_u\right) \qquad (2)$$

where the superscript, *, and subscript, u, identify, respectively, the variables at the reference pressure level and at the top of the model atmosphere. Furthermore,

$$\beta = \left[1 - \sigma^* \left(p_s-p_u\right)/\left(p^*-p_u\right)\right] \left(p_s-p^*\right)^{-3} \qquad (3)$$

where the subscript, s, refers to quantities measured at the surface. We differentiate (2) with respect to time. If

$$\alpha = \sigma^*/p^*-p_u \qquad (4)$$

and

$$J = \left[3\beta(p-p^*)2+\alpha\right] \left(p-p^*\right) \qquad (5)$$

then we may define two coefficients such that

$$\qquad (6)$$
$$q_3 = \left(p-p^*\right)/Jp$$

and

$$q_4 = J_s/Jp \left(\left(p-p^*\right)/\left(p_s-p^*\right)\right)4 \qquad (7)$$

for $p>p^*$, and

$$\qquad (8)$$
$$q_3 = 1/\alpha p = \left(p^*-p_u\right)/\left(\sigma^* p\right)$$

and

$$q_4 = 0 \qquad (9)$$

for $p \leq p^*$.

The thermodynamic equation in the nonlinear sigma coordinates is, upon substitution for the omega-term,

$$\partial T_w/\partial t + m \left(u \, \partial T_w/\partial x + v \, \partial T_w/\partial y\right) + \dot{\sigma} \, \partial T_w/\partial \sigma - RT\omega/C_p \left[q_3\dot{\sigma}+q_4\omega_s\right] - Q/C_p = 0 \quad (10)$$

Here the subscript, w, refers to the whole temperature, $T_w = T_R + T$ where T_R is a reference temperature for the layer and is always in hydrostatic balance and T is the departure from the reference temperature that is subject to adjustment within the variational model. Substitution for the whole temperature retrieves the thermodynamic equation in the adjustable part of the temperature,

$$\partial T/\partial t + m \left(u\, \partial T/\partial x + v\, \partial T/\partial y\right) + \dot{\sigma}\, \partial T/\partial \sigma$$

$$- R/C_p \left(T_R + T\right) \left[q_3\dot{\sigma} + q_4\omega_s\right] + \dot{\sigma}\, \partial T_R/\partial \sigma - Q/C_p = 0 \qquad (11)$$

We next nondimensionalize the thermodynamic equation. Letting

$$\text{Let } u = U\, u', \quad v = Uv', \quad \Delta t = L/C\; \Delta t' \quad \Delta x = L\Delta x'$$

$$T_R = \theta\, T_R' = gH/R\; T_R', \quad \Delta T = \left(gH/R\right)\left(F/R_o\right)\delta T' \quad p = p' \qquad (12)$$

$$\dot{\sigma} = C/L\; \dot{\sigma}' \quad \omega_s = PC/L\; \omega_s'$$

and dividing through by $(C/L)(gH/R)(F/R_o^2)$, the nondimensionalized thermodynamic equation with primes suppressed is,

$$R_o \left[\partial T/\partial t + m \left(u\, \partial T/\partial x + v\, \partial T/\partial y\right) + \dot{\sigma}\, \partial T/\partial \sigma + \dot{\sigma}\, \left(R_o/F\right)\, \partial T_R'/\partial \sigma\right]$$

$$- R_o R/C_p \left(\left(R_o/F\right) T_R' + T\right)\left(q_3\dot{\sigma} + q_4\omega_s\right) - \left[LRR_o^2/CgHF\right] Q/C_p = 0 \qquad (13)$$

Dividing by the additional R_o renders (13) into the same order of magnitude as the other dynamic equations of MODEL II. In addition, it can be shown that the two terms that include T_R combine to form the static stability, σ_σ

$$(14)$$

$$\text{Define } \sigma_\sigma = R_o^2/F \left[\partial T_R'/\partial \sigma - q_3\, R/C_p\, T_R'\right]$$

Therefore, the thermodynamic equation reduces to

$$R_o \left[\partial T/\partial t + m \left(u\, \partial T/\partial x + v\, \partial T/\partial y\right) + \dot{\sigma}\, \left(\partial T/\partial \sigma - \left(R/C_p\right) Tq_3\right)\right] + \dot{\sigma}\, \sigma_\sigma$$

$$- \left[LRR_o^2/CgHF\right] Q/C_p - R_o q_4 \omega_s \left(R/C_p\right)\left(\left(R_o/F\right) T_R + T\right) = 0 \qquad (15)$$

Next, the thermodynamic equation is converted to finite differences and expressed compatible with the Arakawa D-grid finite difference template

developed for MODEL I (Achtemeier, et al., 1986). Fig. 1 shows the template with the locations of the variables that appear in the thermodynamic equation. Note that the local tendency of the temperature has been defined as the dependent and adjustable variable, E_T. The finite difference version of the thermodynamic equation is,

$$R_0 \left[E_T + m^{-xy} u^{-x} T_x^{\sigma--y} + m^{-xy} v^{-y} T_y^{\sigma--x} + \dot{\sigma} \left(T_\sigma^{--xy\sigma} - (R/C_p) q_3^{--xy} T^{--xy} \right) \right]$$
$$+ \sigma_\sigma \dot{\sigma} - R_0 q_4^{--xy} \omega_s R/C_p \left((R_0/F) T_R^- + T^{-xy} \right) - \left[LRR_0^2/CgHF \right] Q/C_p = 0 \tag{16}$$

where the various overbar averages are defined in Achtemeier, et al., (1986).

3. MODEL II: The Variational Equations

The variational analysis melds satellite data with conventional data at the second stage of a two-stage objective analysis. All data are gridded independently in the first stage and are combined in the second stage. The gridded observations to be modified are meshed with the dynamic constraints through Sasaki's (1970) variational formulation which requires the minimization of the integrand of an adjustment functional. Now each element of the integrand, whether the variable to be adjusted, which appears as a least squares term, or the constraint to be satisfied, which enters the formulation through a Lagrange multiplier, is entered as a linear combination. Therefore, it is not necessary to reproduce the full derivation of MODEL I in combination with the thermodynamic equation in order to get MODEL II. Each element can enter the variational formulation separately, the variational terms derived, and the results combined later to form the final set of Euler-Lagrange equations. It is, therefore, necessary to perform the variational operation only upon the thermodynamic equation and the temperature tendency (the only

new dependent variable) and add the results to the appropriate adjustment equations.

Let,

$$\text{Let } I = 2\lambda_5 m_5 + \pi_8 \left(E_T - E_T^o\right)^2 \tag{17}$$

where π_8 is the precision modulus weight for the temperature tendency. Performing the variations upon each of the dependent variables that appear in the thermodynamic equation yields the following terms to be added to the respective Euler-Lagrange equations:

$$\text{ref } \dot{\sigma} \qquad \delta E_T = \pi_8 \left(E_T - E_T^o\right) + R_o \lambda_5 = 0 \tag{18}$$

$$\text{ref } u \qquad \delta u = \delta/u \left[R_o \, \overline{m}^{-xy-x\sigma--y} \, u \, T_x\right] = R_o \, \overline{m}^{-y} \, \overline{\lambda_5 T_x}^{-x\sigma} \tag{19}$$

$$\text{ref } v \qquad \delta v = \delta/v \left[R_o \overline{m}^{-xy-y\sigma--y} \, v \, T_y\right] = R_o \, \overline{m}^{-x} \, \overline{\lambda_5 T_y}^{-yo} \tag{20}$$

$$\text{ref } \dot{\sigma} \qquad \delta\dot{\sigma} = R_o \lambda_5 \left(\overline{T_\sigma}^{--xy\sigma} - (R/C_p) \, \overline{q_3}^{--xy} \, \overline{T}^{-xy}\right) + \lambda_5 \, \sigma_\sigma \tag{21}$$

$$\text{ref } T \qquad \delta\overline{T} = -R_o \left(\overline{m}^{-x} \, \overline{u \, \lambda_5}^{-x--\sigma^y}\right)_x - R_o \left(\overline{m}^{-y-y--\sigma^x} v \, \lambda_5\right)_y - R_o \left(\overline{\sigma\lambda_5}^{---xy\sigma}\right)_\sigma$$
$$- R_o \, R/C_p \left[\overline{\sigma\lambda_5}^{---xy} q_3 + \overline{w_s \lambda_5}^{----xy} q_4\right] \tag{22}$$

Table 1 summarizes the modifications to existing MODEL I variational equations necessary to implement MODEL II. Also included are two new equations, the latter being the thermodynamic equation. This brings to 13 the number of linear and nonlinear equations to be solved.

4. MODEL II: Preliminary Results.

The purpose of this section is to demonstrate that MODEL II performs as predicted by theory. In our evaluation of the variational assimilation models,

we have used three diagnostic criteria which have found use in the verification of diagnostic analyses (Krishnamurti, 1968; Achtemeier, 1975; Achtemeier, et al., 1988; Otto-Bliesner, et al., 1977). These criteria are measures of, first, the extent to which the assimilated fields satisfy the dynamical constraints, second, the extent to which the assimilated fields depart from the observations, and third, the extent to which the assimilated fields are realistic as determined by pattern recognition. The last criterion requires that the signs, magnitudes, and patterns of the hypersensitive vertical velocity and local tendencies of the horizontal velocity components be physically consistent with respect to the larger scale weather systems. Since the purpose of this preliminary study is to evaluate MODEL II as an addition to MODEL I, we will concentrate this report on the first criteria which has successfully revealed that MODEL I, as is currently formulated, does not suffice when combined into MODEL II.

This data assimilation model is derived through the variational method of undetermined Lagrange multipliers (Sasaki, 1970). The strong constraint formalism requires that the dynamical constraints; the nonlinear horizontal momentum equations, the hydrostatic equation, and an integrated form of the continuity equation, be satisfied exactly (to within truncation). Therefore, it is appropriate that the first evaluation of the variational model determine whether indeed the adjusted fields of meteorological variables are solutions of these physical equations.

In solving the Euler-Lagrange equations, we substituted observed or previously adjusted variables into the nonlinear terms and other terms that are products with the Rossby number or are higher order

terms and treated these terms as forcing functions. This approach made the linearized equations easier to solve but several cycles with the forcing terms updated with newly adjusted variables were required for the method to converge to a solution.

In order to determine if the method indeed converges to a solution, it is necessary to average adjusted variables over two successive cycles and reintroduce them into the dynamic constraints. The residuals are computed as remainders of algebraic sums of individual terms of each constraint. The root-mean-squares (RMS) of these differences (Glahn and Lowry, 1972) vanish (constraint satisfaction) when variables at two successive cycles are unchanged. A measure of the magnitude of adjustment required to bring the initial gridded meteorological fields into variational balance is the difference between the initial RMS values (initial unadjustment) obtained by substituting unadjusted variables directly into the dynamic equations and the RMS values at each cycle. Upon dividing by the initial RMS values, the convergence at each cycle can be expressed as percent reduction of the initial unadjustment.

The performance of MODEL II is assessed through the percentage reductions in the RMS differences from the initial unadjustments through the first four cycles of the solution sequence. The calculations are done for the eight adjustable levels in the model. Table 2 shows the percentages for the two nonlinear horizontal momentum equations. These results compare favorably with the MODEL I percentage residual reductions. The initial unadjustments are approximately halved at each cycle to about 90 percent after four cycles.

The percentage reductions of the initial unadjustment for the integrated continuity and hydrostatic equations are shown in Table 3. The RMS differences for the integrated continuity equation are reduced by from 96 to 99 percent at the second cycle and improve slowly to near 100 percent by the fourth cycle. These improvements are, of course, dependent upon the magnitudes of the initial unadjustment. We set the initial vertical velocity to zero. Then the initial unadjustment is equal to the divergence integrated upward. The MODEL I cyclical solution order subjects the adjusted velocity components to a second adjustment to satisfy the integrated continuity equation. In this case, the averages of the adjusted velocity components are just averages of two solutions of the integrated continuity equation. Therefore the unadjustment should approach zero by the second cycle.

The initial unadjustments for the hydrostatic equation at levels 4 through level 8 are halved at each cycle and the percentage reduction increases to near 94 percent by the fourth cycle. Convergence is much slower at levels 1 and 2. There is a 65 percent reduction in the initial unadjustment at the second cycle at level 2. There is no change during the third cycle and a slight increase in the initial unadjustment is observed at cycle 4. Given that the only difference between the adjustments presented here and the adjustments presented for MODEL I is the introduction of the fifth constraint, we are led to suspect that the differences are the result of deleterious impacts by the thermodynamic equation.

Table 4 gives the percentage reductions of the initial unadjustment for the thermodynamic equation. Negative percentages occur where the RMS differences exceed the initial unadjustment. Table 4 shows that the initial

unadjustment was reduced by nearly 90 percent by the fourth cycle at levels 2 and 9. At the remaining levels, first cycle reductions of from 48 to 63 percent were followed by increases in the RMS differences that by the fourth cycle exceeded the initial unadjustment at levels 6 and 7.

Further analysis of the behavior of the convergence of MODEL II has revealed the following:

1. The breakdown in the assimilation is almost exclusively in temperature. The initial unadjustments in the horizontal momentum equations and the continuity equation are reduced as was done with MODEL I. Only the first two levels in the hydrostatic equation show any response to the temperature unadjustment and this is somewhat unexpected given that the most severe departures from convergence in the thermodynamic equation occur at higher levels. Therefore, the breakdown appears confined to some terms in the thermodynamic equation rather than in the variational approach to dynamical data assimilation.

2. The patterns of winds and heights generated by MODEL II are unchanged from the winds and heights generated by MODEL I. This pattern analysis is further revealing that the breakdown in convergence in MODEL II is largely confined to the thermodynamic equation.

3. The initial unadjustment in the thermodynamic equation was found to be approximately an order of magnitude larger than the initial unadjustments for the other dynamic constraints and was approximately two orders of magnitude larger in the stratosphere. Although this is not the cause for the breakdown in convergence, it does show that a gross imbalance existed in the initial

gridded fields of meteorological variables when those variables were substituted into the thermodynamic equation.

4. Analysis of the patterns of the residuals remaining after the fourth pass found that they were almost identical to, and mostly caused by, the patterns of vertical velocity.

Our analysis of the large RMS differences in the thermodynamic equation remaining after four cycles reveals the following concerning how the initial and adjusted vertical velocity adversely impacted upon the analyses. First, the initial vertical velocity was calculated kinematically and subjected to the variational adjustment by O'Brien (1970). This method can transfer error from the lower levels into the upper levels of the troposphere and generate large and noisy vertical velocity patterns there. Furthermore, there is no consideration given for the change in static stability between the troposphere with its relatively large vertical velocities and the stratosphere with its relatively small vertical velocities. The kinematic vertical velocities were unrealistically large in the stratosphere and, when coupled with the large static stability, produced large and uncompensated terms in the thermodynamic equation. Therefore, the magnitudes of the initial unadjustments were approximately two orders of magnitude larger than were the initial unadjustments for the other dynamical constraints.

Second, further theoretical analysis has revealed that the adjustment for the divergent part of the wind is the "weak link" in this variational assimilation model. First order terms that contain the divergence adjustment cancel out in the cyclical solution formulations. The divergence adjustment

must then be carried in second order terms and through other variables. Our solution for this problem has been to require the adjusted horizontal velocity components to satisfy the continuity equation constraint after each cycle, a variational model within a variational model, then allow for the second order terms and the readjusted velocity components to "nudge" the solution toward the desired dynamic balance. The result was that the RMS differences grew after the first cycle when the vertical velocity was released to converge slowly toward another equilibrium.

5. Coupling the Vertical Velocity in MODEL I.

In this section, we propose solutions for the vertical velocity related problems of very large initial unadjustments for the thermodynamic equation and the buildup of RMS differences in MODEL II.

The solution for the problem of very large initial unadjustments in the thermodynamic equation is the implementation of a blended vertical velocity algorithm such as the variational method presented by Chance (1986). This method, developed as part of this variational assimilation project but not included in the version of MODEL II evaluated as part of this study, blends the divergence of the horizontal wind with the vertical velocity calculated from the adiabatic method. The relative weighting given the horizontal and the vertical velocity is a function of the stability, relative humidity, and satellite observed cloud cover. The divergence of the horizontal wind receives the greatest weight when the conditions of low stability, near saturation, or dense cloud cover at levels with near saturation prevail. The adiabatic vertical velocity receives greatest weight at locations where stability is

high. Division by large stability reduces the magnitude of the vertical velocity in the stratosphere and forces the vertical velocity to near zero at the tropopause rather than at the arbitrarily defined top of the model domain.

Preliminary studies with the blended vertical velocity show that large magnitude centers of either sign developed by the kinematic method in the upper troposphere and lower stratosphere are reduced or eliminated. Therefore the large initial unadjustments that exist because of the use of the kinematic vertical velocities will be reduced or eliminated also.

The solution for the problem of buildup of RMS differences in MODEL II is to reformulate the MODEL I variational equations so that the solution sequence will better couple the vertical velocity with the dynamic adjustment. Achtemeier, et al. (1986) and Achtemeier and Ochs (1988) have shown that the derivations in MODEL I required to reduce the number of dependent variables and equations to a single diagnostic equation in geopotential cancel out the zero order divergence adjustment terms. The adjustment of the divergent part of the wind is therefore forced into higher-order nonlinear terms which do not sufficiently impact upon the final adjustment to bring about compatibility with the continuity equation. This compatibility is established through the second variational step.

Our analysis of the performance of MODEL II reveals that the second variational step must be eliminated and the coupling of the vertical velocity with the remainder of the adjusted variables must be part of a single variational model. It was found that the divergent part of the wind retrieved from the first step adjustment is a function of the nonlinear terms of the

horizontal momentum equations (see Achtemeier, et al., (1986) and Achtemeier and Ochs (1988). If F_5 represents the nonlinear terms of the u-component equation and F_6 represents the nonlinear terms of the v-component equation, then the horizontal momentum equations can be expressed as

$$m_1 = -v + \partial\phi/\partial x + F_5 = 0 \qquad (23)$$

$$m_2 = u + \partial\phi/\partial y + F_6 = 0 \qquad (24)$$

Forming the divergence from (23) and (24) and integrating through the depth of the analysis domain gives

$$\int (u_x + v_y)\, d\sigma = -\int (F_{6x} - F_{5y})\, d\sigma = 0 \qquad (25)$$

It is apparent that (25) is an integrated form of the vorticity equation. The constraint upon the divergent part of the wind, and hence the vertical velocity, that must be satisfied in order for all MODEL I dynamic constraints to be satisfied without the inclusion of an ancillary variational solution for the integrated continuity equation is as follows. A particular solution of the vorticity equation must integrate to zero at the top of the model domain and the divergent component of the same adjusted wind field must also satisfy the integrated continuity equation.

The above stated condition has been incorporated into the variational

formalisms for MODEL I and the Euler-Lagrange equations rederived. The results show that major modifications of the MODEL I theory are required to implement the vorticity equation constraint. The reprogramming of MODEL I is currently underway.

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Table 1. Modifications to variational equations in MODEL 1 to obtain MODEL 2.

Variable Referenced	Existing Function	New Terms to be Added
u	F_1	$R_o \, \overline{m}^{-y} \lambda_5 \overline{\overline{T}}_x^{x\sigma}$
v	F_2	$R_o \, \overline{m}^{-x} \lambda_5 \overline{\overline{T}}_y^{y\sigma}$
$\dot{\sigma}$	F_3	$\lambda_5 \sigma_\sigma + R_o \lambda_5 \left(\overline{\overline{T}}_\sigma^{xy\sigma} - (R/C_p) \, \overline{\overline{q_3}}^{xy} \, \overline{\overline{T}}^{xy} \right)$
$\overline{T}$	Eq 34 p 39 Final Report 1986	$F_8' = -\left[\left(\overline{m}^{-x} \overline{u}^{-x} \overline{\lambda_5}^{--\sigma} \right)_x^{y} + \left(\overline{m}^{-y} \overline{v}^{-y} \overline{\lambda_5}^{--\sigma} \right)_y^{x} + \left(\sigma \overline{\overline{\lambda_5}}^{xy\sigma} \right)_\sigma + (R/C_p) \left(\sigma \overline{\overline{\lambda_5}}^{xy} q_3 + \omega_s \overline{\overline{\lambda_5}}^{xy} q_4 \right) \right]$
$\overline{T}$	F_8 Eq 47 p 41 Final Report 1986	F_8'/Y
$\dot{\sigma}$	New Equation	$\lambda_5 = -\,\Pi_8/R_o \left(E_T - E_T^o \right)$
$\dot{\sigma}$	New Equation	$E_T = -\left[\overline{m}^{-xy} \overline{u}^{-x\sigma} \overline{\overline{T}}_x^{-y} + \overline{m}^{-xy} \overline{v}^{-y\sigma} \overline{\overline{T}}_y^{-x} + \dot{\sigma} \left(\overline{\overline{T}}_\sigma^{xy\sigma} - (R/C_p) \, \overline{\overline{q_3}}^{xy} \, \overline{\overline{T}}^{xy} \right) + \sigma_\sigma/R_o \, \dot{\sigma} - \overline{\overline{q_4}}^{xy} \omega_s \, (R/C_p) \left(R_o/F \, \overline{T}_R + \overline{T}^{xy} \right) \right]$

Table 2. Percent reduction of the initial unadjustment in the horizontal momentum equations after 4 cycles.

Cycle No.	2	3	4	Level 5	6	7	8	9
				u-component				
0	0	0	0	0	0	0	0	0
1	54	54	52	51	50	50	51	51
2	81	78	77	75	74	75	76	76
3	92	89	87	86	86	87	87	87
4	94	93	90	89	91	91	90	90
				v-component				
0	0	0	0	0	0	0	0	0
1	54	53	52	53	51	51	50	50
2	78	80	77	80	77	76	76	73
3	88	89	87	90	88	88	87	84
4	93	92	91	92	91	91	91	88

Table 3. Percent reduction of the initial unadjustment in the integrated continuity and hydrostatic equations after 4 cycles.

Cycle No.	2	3	4	Level 5	6	7	8	9
				Integrated Continuity				
0	0	0	0	0	0	0	0	0
1	-	-	-	-	-	-	-	-
2	97	98	98	99	99	99	99	99
3	96	98	98	99	99	99	99	99
4	96	98	99	99	99	99	99	99
				Hydrostatic				
0	0	0	0	0	0	0	0	0
1	51	50	50	50	50	50	50	50
2	73	65	75	75	75	75	75	75
3	83	65	88	88	88	88	88	88
4	86	62	94	94	94	94	94	94

Table 4. Percent reduction of the initial unadjustment
in the thermodynamic equation after 4 cycles.

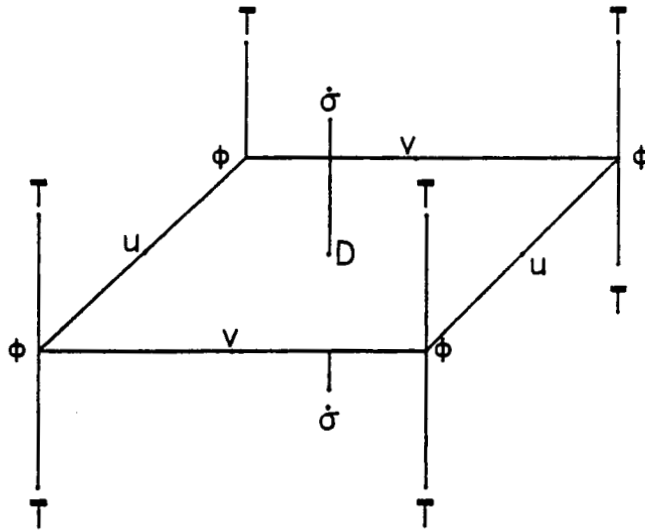| Cycle No.	Level 2	3	4	5	6	7	8	9
			Thermodynamic Equation					
0	0	0	0	0	0	0	0	0
1	54	60	62	63	61	63	63	48
2	81	80	74	55	24	39	76	72
3	89	73	61	32	-12	9	62	83
4	88	65	50	14	-38	-12	49	89

Fig. 1. The grid template for the variational assimilation model.